The tree property at $\aleph_{\omega+2}$ with a finite gap

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Let κ be an infinite regular cardinal. The tree property at κ is a compactness principle which says that every κ-tree has a cofinal branch.

Obtaining the tree property at the double successor of an infinite regular cardinal κ is relatively easy and only a weakly compact cardinal is required ("Mitchell forcing"). The situation is more complex when we wish to get this result at the double successor of a singular strong limit cardinal κ since we need to ensure the failure of SCH at κ.

In this talk we will discuss the important case of $\aleph_\omega$ and show that if κ is a certain large cardinal (not too large), and $1 < n < \omega$ is fixed, then there is a forcing $\mathbb{P}$ such that the following hold in $V^{\mathbb{P}}$:

- $\kappa = \aleph_\omega$ is strong limit,
- $2^{\aleph_\omega} = \aleph_{\omega+n}$, and
- The tree property holds at $\aleph_{\omega+2}$.

The forcing $\mathbb{P}$ is a combination of several subforcings which first prepare the universe and then use a combination of the Mitchell forcing and the Prikry forcing with collapses to force the tree property.